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A Novel Chaotic Neural Network Architecture

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1. Introduction

The basic premise of this research is that deterministic chaos is a powerful mechanism for the storage and retrieval of information in the dynamics of artificial neural networks. Substantial evidence has been found in biological studies for the presence of chaos in the dynamics of natural neuronal systems [1-3]. Many have suggested that this chaos plays a central role in memory storage and retrieval [1,4-6]. Indeed, chaos offers many advantages over alternative memory storage mechanisms used in artificial neural networks. One is that chaotic dynamics are significantly easier to control than other linear or non-linear systems, requiring only small appropriately timed perturbations to constrain them within specific Unstable Periodic Orbits (UPOs). Another is that chaotic attractors contain an infinite number of these UPOs. If individual UPOs can be made to represent specific internal memory states of a system, then in theory a chaotic attractor can provide an infinite memory store for the system. In this paper we investigate the possibility that a network can self-select UPOs in response to specific dynamic input signals. These UPOs correspond to network recognition states for these input signals.

2. Controlling Chaos

One of the surprising features of chaotic systems is the ease with which they can be controlled. Several methods have been developed for the control of chaotic systems over recent years [7-9]. We believe that the feedback method of control is most appropriate in the context of chaotic neural networks. The reasons for this are based on the biological plausibility of the control method. First, the delay feedback method does not rely on *a priori* knowledge of the local dynamics of the attractor around the UPO to be stabilised. It seems very unlikely that biological neuronal networks have this kind of detailed knowledge of their own dynamics. Second, the delayed feedback method does not specify which UPO is to be stabilised, it simply specifies the period of the required orbit. This suggests an element of self-organisation which is biologically appealing. Third, delays in signal transmission are inherent in all biological neuronal networks. The nerve impulse takes a period of time to travel the length of the axon to its target neurons which, in turn, take time to summate their inputs and produce their response. Fourth, when applied to neural networks, the feedback control method amounts to delayed inhibition, which is a common element

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of many natural neuronal systems. Consequently, the delayed feedback method is considered to be best suited to the control of chaos in neural networks.

The delayed feedback method of chaos control used here is modified from a method originally proposed by Pyragas [8]. Pyragas's method was applied to continuous time systems which have a measurable output variable, say *y(t)*, and an input signal, *F*(*t*):

$$\frac{dy}{dt} = P(y, \mathbf{x}) + F(t) \qquad \frac{d\mathbf{x}}{dt} = \mathbf{Q}(y, \mathbf{x}) \qquad F(t) = k\left[y(t) - y(t-\tau)\right] \tag{1}$$

Where $P(y,\mathbf{x})$ and $\mathbf{Q}(y,\mathbf{x})$ which govern the chaotic dynamics of the system, and $\mathbf{x}$, which denotes all of the remaining system variables, are assumed to be unknown. When the control signal $F(t)$ is zero, the system (1) is governed by a chaotic attractor.

$F(t)$ attempts to nudge the system back to a state in which output variable y repeats a value it had at the earlier time specified by the delay ⓿. In this way, $F(t)$ encourages the system to follow a periodic trajectory with periodicity ⓿. As the system approaches the periodic trajectory, $F(t)$ will become very small. UPOs of varying periods and periodicities can be controlled by this method.

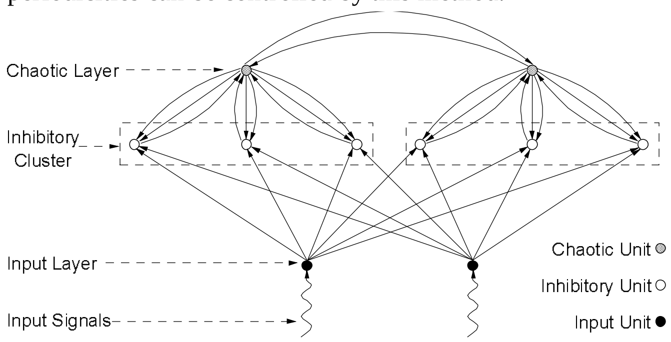

Figure 1 Overview of the Chaotic Neural Network Architecture

Chaotic Neural Network Model

We have designed a network model which has a neural implementation of the delayed feedback method of chaos control described in the previous section. An overview of the network's architecture is illustrated in Figure 1. The network has three layers. The first layer consists of a set of units which receive dynamic input signals. This input layer is fully connected to the units in the second layer which is made up of clusters of inhibitory units. All of the units in a particular inhibitory cluster are connected to the same unit in the chaotic layer. Each unit in the chaotic layer is fully connected with the other units in that layer via lateral connections.

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Each unit in the Chaotic Layer is governed by the following discrete time equations which have been modified from [10]:

$$y_i(t+1) = \omega y_i(t) - \alpha f(y_i(t)) + a + \sum_{j \neq i}^{M} w_{ij} x_j(t - \tau_{ij}) + \sum_{j=1}^{N} k_{ij} z_j(t) \quad (2)$$

$$x_i(t+1) = f(y_i(t+1)) \quad (3)$$

where $y_i(t)$ is the internal state of unit i at time t, ω ($0 < \omega < 1$), α ($\alpha > 0$) and a are parameters of the Aihara model, M is the number of units in the Chaotic Layer and N is the number of units in each inhibitory cluster. The connections between units in the chaotic layer have a weight denoted by w_{ij} and a time delay denoted by τ_{ij}. $x_i(t)$ is the output activation of chaotic unit i at time t. The weights of connections from the inhibitory units to the chaotic units are denoted by k_{ij}, and the activation of chaotic unit j at time t is denoted by $z_j(t)$. $f(y)$ is given by:

$$f(y) = \frac{1}{1 + e^{-y/\varepsilon}} \quad (4)$$

When the Inhibitory units are inactive for a period of time (i.e. the last term of equation (2) is zero for a number of time steps) the dynamics of equations (2) and (3) are governed by a chaotic attractor.

The units in the inhibitory layer are divided into clusters, with one cluster for each of the units in the chaotic layer. The purpose of each inhibitory cluster is to apply feedback control to stabilise the associated chaotic unit into a UPO. Each inhibitory unit receives two inputs from the chaotic unit associated with that cluster. The first input is $x_i(t)$, which the activation of the chaotic unit i at time t. The second is $x_i(t - \tau_{ji})$, which is the activation of chaotic unit i at time $t - \tau_{ji}$, where τ_{ji} is a randomly selected time delay. An important feature of the architecture is that each inhibitory unit has a different randomised time delay connection with the associated chaotic unit.

The inhibitory units within a cluster compete with each other for the right to attempt to control the associated chaotic unit. At each time step, only one unit from the cluster will win the right to apply control to the chaotic unit. The competition is based on the value of $h(t)$, which is given by

$$h_j(t) = \left| f\left(\sum_{k=1}^{L} w_{j,k} I_k(t)\right) - x_i(t - \tau_{j,i}) \right| \quad (5)$$

where L is the number of input units, and $I_k(t)$ is the activation of the kth input unit at time t. The inhibitory unit with the smallest value for $h(t)$ at that time step wins and has an activation value calculated by the following equation:

$$z_j(t+1) = x_i(t) - x_i(t - \tau_{j,i}) \quad (6)$$

All other units in that inhibitory cluster have their activation values set to zero for that time step. The value $z(t)$ is the control required to stabilise an orbit in the associated chaotic unit with period τ_{ij}.

The input layer is fully connected to the units in the inhibitory layer. These connections are weighted and instantaneous (i.e. there are no time delays). In the model presented in this paper all the weights from the input layer to the inhibitory

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layer are set to 1. The activations of the input units at each time step are governed entirely by the input sequences consisting of discrete values in the range 0 to 1.

3 Experimental Results

In this section we present some preliminary results from two experiments with a network consisting of 1 input unit, 3 units in each inhibitory cluster, and 4 chaotic units. In each experiment, the network was iterated for 200 time steps before the input signals were activated. The network was then iterated for a further 200 times steps. The input patterns used were:

Input sequence (a) 1.0, 0.5

Input sequence (b) 1.0, 0.75, 0.5, 0.25, 0.01, 0.25, 0.75

The first experiment used input sequence (a) and the second used input sequence (b). When the input was initiated at t=200, consecutive values of the input sequence would be set as the activation value for the input unit on each time step, repeating the sequence as many times as necessary (i.e. for sequence (a) this would be: *I*(1) = 1.0, *I*(2) =0.5, *I*(3) =1.0, *I*(4) =0.5, etc.) Figures 3 and 4 show the times series of the activations of some of the units during the first experiment.

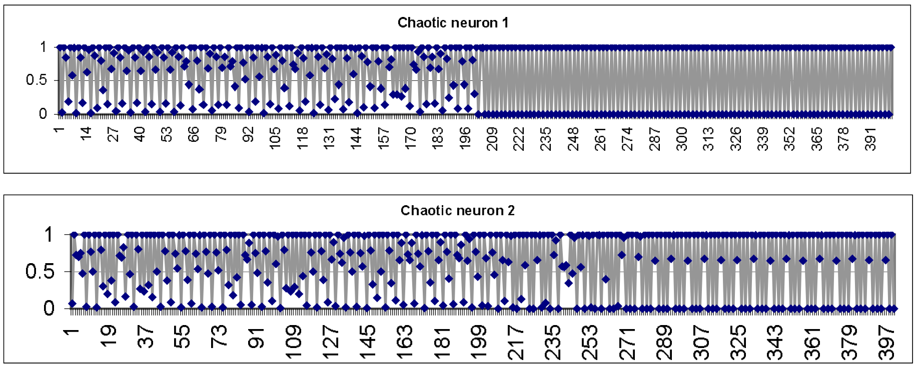

Figure 3 The activations of 2 units from the Chaotic Layer

It was important to ensure that the temporal pattern of activation for each unit in the chaotic layer was truly chaotic in the absence of an input sequence. In each experiment, the network was iterated 200 times without the presence of input to generate a sufficiently long data series to demonstrate the presence of chaos. The average Lyapunov Exponent for the activation of each chaotic unit during this first phase was positive, indicating that they were truly chaotic (LE_1 = 0.252, LE_2 = 0.28, LE_3 = 0.255, LE_4 = 0.19). The input sequence (a) was initiated at t = 200. Figure 3 shows that chaotic unit 1 was stabilised into periodic orbits almost instantaneously. Chaotic unit 2 took longer to come under control. The average Lyapunov Exponent for all 4 chaotic units in the input phase of the experiment are negative, indicated non-chaotic activations (Table 1).

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A significant feature of these results is that each of the chaotic units is stabilised to orbits with different periods (see Table 1). This corresponds to a distributed recognition state for this input sequence. This recognition state (or representation) is distributed both spatially and temporally: it is distributed spatially because of the pattern of periods of stabilised orbits is distributed across the chaotic layer; it is distributed temporally because the representation consists of stabilised periodic orbits, which are temporal patterns of activation.

Figure 4 shows the activations of each of the units in the inhibitory layer during this first experiment. During the first 200 time steps when there is no input to the network, these units are inactive, allowing the chaotic layer to follow its chaotic attractor. When the input sequence is activated at t = 200, the inhibitory units begin to exert control on the chaotic layer. It should be noted that this control is shared between the three units in each of the clusters.

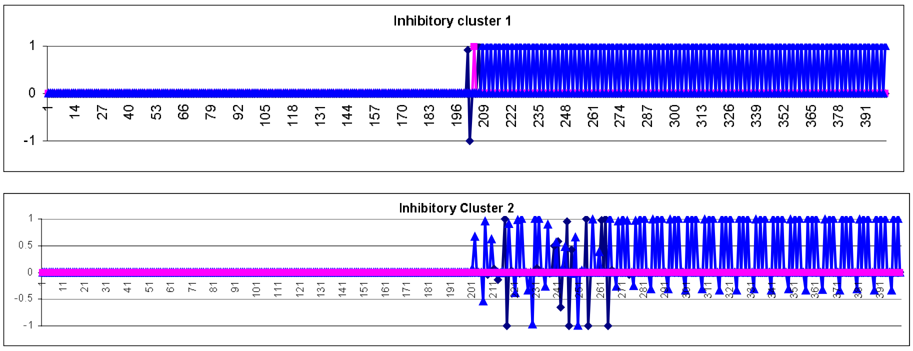

Figure 4 The activations of the units in 2 of the inhibitory clusters

The second experiment was carried out on the same network (i.e. identical in terms of structure, weights and time delays) using input sequence (b). Table 1 shows the results from this experiment. In this second experiment the chaotic layer was stabilised to a different set of periodic orbits for this input sequence than were stabilised for the first input sequence. In this way the network has produced a different response to different input patterns.

4. Conclusion

We have constructed a chaotic neural network which is capable of differentiating between two dynamic input signals by producing different dynamic responses. The chaotic layer is responsible for producing these dynamic responses or “representations” of the input. The representations consists of temporally and spatially distributed patterns across the chaotic units. The temporal aspect of the representation is the Unstable Periodic Orbits (UPOs) which the chaotic units are constrained to follow by the inhibitory layer. There are an infinite number of UPOs embedded within the attractors of chaotic systems. Consequently, there is the potential of developing networks with extremely large memory capacities and where each memory is efficiently embedded in the dynamics of the network.

The results presented here are very provisional and only provide an indication of what may be possible with chaotic neural networks of this kind. Future research will need to assess this network's ability to generalise and correctly classify unfamiliar input pattern sequences. Although this is a fixed weight network, it does have inherent adaptability thought the control of chaos mechanism in the inhibitory layer. This mechanism is adaptive both in its response to the input patterns because of the competitive element in each cluster and in its control of the chaotic layer, through the selection of an appropriate strength of delayed feedback.

		Chaotic unit 1	**Chaotic unit 2**	**Chaotic unit 3**	**Chaotic unit 4**
Input squence 1:	**Period of Orbit**	2	8	2	18
	Lyapunov Exponent	-0.61	-0.50	-0.46	-0.30
Input sequence 2:	**Period of Orbit**	24	24(8)	6	2
	Lyapunov Exponent	-0.18	-0.21	-0.19	-0.48

Table 1 Outline of the results from both experiments

References

[1] W.J. Freeman and J.M. Barrie. Chaotic oscillations and the genesis of meaning in cerebral cortex. In: *Temporal Coding in the Brain*, eds. G. Buzsaki, R. Llinas, W. Singer, A. Berthoz, and Y. Christen. Berlin: Springer-Verlag, 1994.pp. 13-37.

[2] M.R. Guevara, L. Glass, M.C. Mackey, and A. Shrier, Chaos in Neurobiology *IEEE Transctions on Systems, Man, and Cybernetic*, vol. 13, pp. 790-798, 1983.

[3] A. Babloyantz and C. Lourenco, Brain Chaos and Computation *International Journal of Neural Systems*, vol. 7, pp. 461-471, 1996.

[4] C. Lourenco and A. Babloyantz, Control of spatiotemporal chaos in neural networks *International Journal of Neural Systems*, vol. 7, pp. 507-517, 1996.

[5] I. Tsuda, A New Type of Self-Organization Associated with Chaotic Dynamics in Neural Networks *International Journal of Neural Systems*, vol. 7, pp. 451-459, 1996.

[6] N.T. Crook, C.H. Dobbyn, and T. olde Scheper. Chaos as a Desirable Stable State of Artificial Neural Networks. In: *Advances in Soft Computing: Soft Computing Techniques and Applications*, eds. R. John and R. Birkenhead. Physica-Verlag, 2000.pp. 52-60.

[7] E. Ott, C. Grebogi, and J.A. Yorke, Controlling Chaos *Physical Review Letters*, vol. 64, pp. 1196-1199, 1990.

[8] K. Pyragas, Continuous control of chaos by self-controlling feedback *Physics Letters A*, vol. 170, 1992.

[9] T. Shinbrot, C. Grebogi, E. Ott, and J.A. Yorke, Using Small Perturbations to Control Chaos *Nature*, vol. 363, pp. 411-417, 1993.

[10] K. Aihara, T. Takabe, and I. Tsuda, Chaotic Neural Networks *Physics Letters A*, vol. 144, pp. 333-340, 1990.